Chapter 9

Rotation

Problems: 3, 4, 12, 30, 32, 41, 52, 71, 84, 108, 114

Think about: 7, 82!, 92, 121!

3 • Starting from rest and rotating at constant angular acceleration, a disk takes 10 revolutions to reach an angular speed *ω*. How many additional revolutions at the same angular acceleration are required for it to reach an angular speed of 2*ω*? (*a*) 10 rev, (*b*) 20 rev, (*c*) 30 rev, (*d*) 40 rev, (*e*) 50 rev?

Picture the Problem The constant-acceleration equation that relates the given variables is $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$. We can set up a proportion to determine the number of revolutions required to double *ω* and then subtract to find the number of additional revolutions to accelerate the disk to an angular speed of 2*ω*.

Using a constant-acceleration equation, relate the initial and final angular velocities to the angular acceleration:

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

or, because $\omega_0^2 = 0$,

$$\omega^2 = 2\alpha\Delta\theta$$

Let $\Delta\theta_{10}$ represent the number of revolutions required to reach an angular speed *ω*:

$$\omega^2 = 2\alpha\Delta\theta_{10} \quad (1)$$

Let $\Delta\theta_{2\omega}$ represent the number of revolutions required to reach an angular speed *ω*:

$$(2\omega)^2 = 2\alpha\Delta\theta_{2\omega} \quad (2)$$

Divide equation (2) by equation (1) and solve for $\Delta\theta_{2\omega}$:

$$\Delta\theta_{2\omega} = \frac{(2\omega)^2}{\omega^2}\Delta\theta_{10} = 4\Delta\theta_{10}$$

The number of *additional* revolutions is:

$$4\Delta\theta_{10} - \Delta\theta_{10} = 3\Delta\theta_{10} = 3(10\,\text{rev}) = 30\,\text{rev}$$

and $\boxed{(c)}$ is correct.

4 • You are looking down from above at a merry-go-round, and observe that is rotating counterclockwise and its rotation rate is slowing. If we designate counterclockwise as positive, what is the sign of the angular acceleration?

Determine the Concept Because the merry-go-round is slowing, the sign of its angular acceleration is $\boxed{\text{negative.}}$

7 • **[SSM]** During a baseball game, the pitcher has a blazing fastball. You have not been able to swing the bat in time to hit the ball. You are now just trying to make the bat contact the ball, hit the ball foul, and avoid a strikeout. So, you decide to take your coach's advice and grip the bat high rather than at the very end. This change should increase bat speed; thus you will be able to swing the bat quicker and increase your chances of hitting the ball. Explain how this theory works in terms of the moment of inertia, angular acceleration, and torque of the bat.

Determine the Concept The closer the rotation axis to the center of mass, the smaller the moment of inertia of the bat. By choking up, you are rotating the bat about an axis closer to the center of mass, thus reducing the bat's moment of inertia. The smaller the moment of inertia the larger the angular acceleration (a quicker bat).

12 • A constant net torque acts on a merry-go-round from startup until it reaches its operating speed. During this time, the merry-go-round's rotational kinetic energy (*a*) is constant, (*b*) increases linearly with angular speed, (*c*) increases quadratically as the square of the angular speed, (*d*) none of the above.

Determine the Concept The work done by the net torque increases the rotational kinetic energy of the merry-go-round. Because $\boldsymbol{K}_{\text{rot}} = \frac{1}{2}\boldsymbol{I\omega}^2$, $\boxed{(c)}$ is correct.

30 • When a turntable rotating at 33 rev/min is shut off, it comes to rest in 26 s. Assuming constant angular acceleration, find (*a*) the angular acceleration. During the 26 s, find (*b*) the average angular speed, and (*c*) the angular displacement, in revolutions.

Picture the Problem Because we're assuming constant angular acceleration; we can find the various physical quantities called for in this problem by using constant-acceleration equations for rotational motion.

(*a*) The angular acceleration of the turntable is given by:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{0 - 33\dfrac{\text{rev}}{\text{min}} \times \dfrac{2\pi\,\text{rad}}{\text{rev}} \times \dfrac{1\,\text{min}}{60\,\text{s}}}{26\,\text{s}} = \boxed{0.13\,\text{rad/s}^2}$$

(*b*) Because the angular acceleration is constant, the average angular speed is given by:

$$\omega_{\text{av}} = \frac{\omega_0 + \omega}{2}$$

Substitute numerical values and evaluate ω_{av}:

$$\omega_{av} = \frac{33\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}}{2} = 1.73\,\text{rad/s} = \boxed{1.7\,\text{rad/s}}$$

(*c*) Using the definition of ω_{av}, find the angular displacement of the turntable as it slows to a stop:

$$\Delta\theta = \omega_{av}\Delta t = (1.73\,\text{rad/s})(26\,\text{s}) = 44.9\,\text{rad} \times \frac{1\,\text{rev}}{2\pi\,\text{rad}} = \boxed{7.2\,\text{rev}}$$

32 • A 12-m-radius Ferris wheel rotates once each 27 s. (*a*) What is its angular speed (in radians per second)? (*b*) What is the linear speed of a passenger? (*c*) What is the acceleration of a passenger?

Picture the Problem We can find the angular speed of the Ferris wheel from its definition and the linear speed and centripetal acceleration of the passenger from the relationships between those quantities and the angular speed of the Ferris wheel.

(*a*) Find ω from its definition:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{27\,\text{s}} = 0.233\,\text{rad/s} = \boxed{0.23\,\text{rad/s}}$$

(*b*) Find the linear speed of the passenger from his/her angular speed:

$$\boldsymbol{v} = \boldsymbol{r\omega} = (12\,\text{m})(0.233\,\text{rad/s}) = \boxed{2.8\,\text{m/s}}$$

Find the passenger's centripetal acceleration from his/her angular speed:

$$\boldsymbol{a}_c = \boldsymbol{r\omega}^2 = (12\,\text{m})(0.233\,\text{rad/s})^2 = \boxed{0.65\,\text{m/s}^2}$$

41 • **[SSM]** Four particles, one at each of the four corners of a square with 2.0-m long edges, are connected by massless rods (Figure 9-45). The masses of the particles are $m_1 = m_3 = 3.0$ kg and $m_2 = m_4 = 4.0$ kg. Find the moment of inertia of the system about the z axis.

Picture the Problem The moment of inertia of a system of particles with respect to a given axis is the sum of the products of the mass of each particle and the square of its distance from the given axis.

Use the definition of the moment of inertia of a system of four particles to obtain:

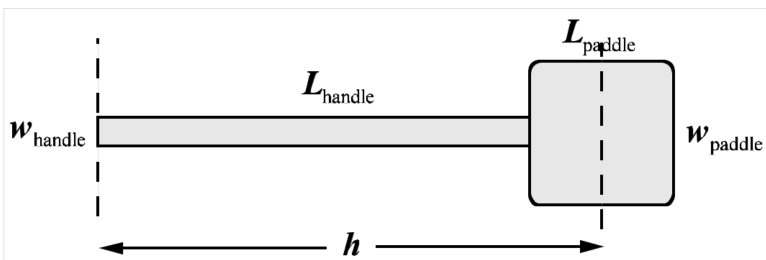
$$I = \sum_i m_i r_i^2$$
$$= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate $I_{z\text{ axis}}$:

$$I_{z\text{ axis}} = (3.0\,\text{kg})(2.0\,\text{m})^2 + (4.0\,\text{kg})\left(2\sqrt{2}\,\text{m}\right)^2 + (4.0\,\text{kg})(2.0\,\text{m})^2 + (3.0\,\text{kg})(0)^2$$
$$= \boxed{60\,\text{kg}\cdot\text{m}^2}$$

52 •• To prevent damage to her shoulders, your elderly grandmother wants to purchase the rug beater (Figure 9-50) with the lowest moment of inertia about its grip end. Knowing you are taking physics, she asks your advice. There are two models to choose from. Model A has a 1.0-m-long handle and a 40-cm-edge-length square, where the masses of the handle and square are 1.0 kg and 0.5 kg, respectively. Model B has a 0.75-m-long handle and a 30-cm-edge-length square, where the masses of the handle and square are 1.0 kg and 0.5 kg, respectively. Which model should you recommend? Determine which beater is easiest to swing from the very end by computing the moment of inertia for both beaters.

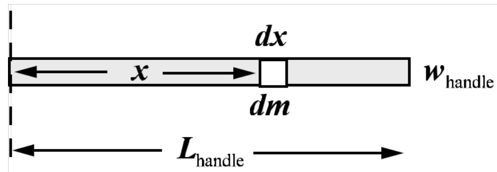
Picture the Problem The pictorial representation models the rug beater as two rectangles of different dimensions. The moment of inertia of the rug beater, about the axis shown to the left, is the sum of the moments of inertia of the handle and the paddle. The rug beater that is easiest for your grandmother to use is the one with the smaller moment of inertia about an axis through the grip end of the handle.



The moment of inertia of the rug beater, with respect to an axis through the end of its handle, is the sum of the moments of inertia of its handle and paddle:

$$I = I_{\text{handle}} + I_{\text{paddle}} \quad (1)$$

The handle of the rug beater is shown to the right. We can apply $I = \int x^2 dm$ to this configuration to derive an expression for I_{handle}.



The moment of inertia of the handle, about an axis through the grip end of the handle, is:

$$I = \int x^2 dm$$

Let t_{handle} represent the thickness of the handle and ρ its density yields:

$$dm = \rho dV = \rho w_{\text{handle}} t_{\text{handle}} dx$$

Substituting for dm yields:

$$I_{\text{handle}} = \int \rho w_{\text{handle}} t_{\text{handle}} x^2 dx$$
$$= \rho w_{\text{handle}} t_{\text{handle}} \int x^2 dx$$

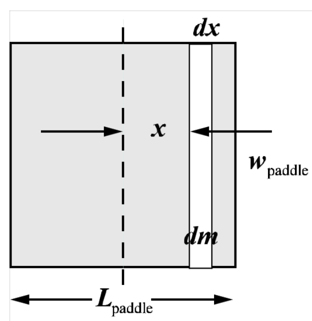
Integrating over the length of the handle yields:

$$I = \rho w_{\text{handle}} t_{\text{handle}} \int_0^{L_{\text{handle}}} x^2 dx$$
$$= \tfrac{1}{3} \rho w_{\text{handle}} t_{\text{handle}} L_{\text{handle}}^3$$

Because the mass of the handle is given by $m_{\text{handle}} = \rho w_{\text{handle}} t_{\text{handle}} L_{\text{handle}}$:

$$I_{\text{handle}} = \tfrac{1}{3} m_{\text{handle}} L_{\text{handle}}^2$$

We can find the moment of inertia of the paddle, relative to an axis through the grip end of the handle, by first finding its moment of inertia with respect to an axis through its center of mass and then applying the parallel-axis theorem.



The moment of inertia, about an axis through the center of mass of the paddle, is:

$$I_{\text{paddle}} = \int x^2 dm$$

The mass of the infinitesimal element of the paddle is given by:

$$dm = \rho dV = \rho w_{\text{paddle}} t_{\text{paddle}} dx$$

Substituting for *dm* yields:

$$I = \int \rho w_{\text{paddle}} t_{\text{paddle}} x^2 dx = \rho w_{\text{paddle}} t_{\text{paddle}} \int x^2 dx$$

Integrating over the length L_{paddle} of the rectangular object yields:

$$I_{\text{paddle}} = \rho w_{\text{paddle}} t_{\text{paddle}} \int_{-\frac{1}{2}L_{\text{paddle}}}^{\frac{1}{2}L_{\text{paddle}}} x^2 dx = \tfrac{1}{12} \rho w_{\text{paddle}} t_{\text{paddle}} L_{\text{paddle}}^3$$

Because the mass of the paddle is given by $m_{\text{paddle}} = \rho w_{\text{paddle}} t_{\text{paddle}} L_{\text{paddle}}$:

$$I_{\text{paddle, cm}} = \tfrac{1}{12} m_{\text{paddle}} L_{\text{paddle}}^2$$

Apply the parallel-axis theorem to express the moment of inertia of the paddle with respect to an axis through the grip end of the handle:

$$I_{\text{paddle}} = I_{\text{cm}} + m_{\text{paddle}} h^2$$

or, because $h = L_{\text{handle}} + \frac{1}{2} L_{\text{paddle}}$,

$$I_{\text{paddle}} = I_{\text{paddle, cm}} + m_{\text{paddle}} \left(L_{\text{handle}} + \tfrac{1}{2} L_{\text{paddle}} \right)^2$$

Substituting for $I_{\text{paddle, cm}}$ yields:

$$I_{\text{paddle}} = \tfrac{1}{12} m_{\text{paddle}} L_{\text{paddle}}^2 + m_{\text{paddle}} \left(L_{\text{handle}} + \tfrac{1}{2} L_{\text{paddle}} \right)^2$$

Substitute for I_{handle} and I_{paddle} in equation (1) to obtain:

$$I = \tfrac{1}{3} m_{\text{handle}} L_{\text{handle}}^2 + \tfrac{1}{12} m_{\text{paddle}} L_{\text{paddle}}^2 + m_{\text{paddle}} \left(L_{\text{handle}} + \tfrac{1}{2} L_{\text{paddle}} \right)^2$$

Substitute numerical values and evaluate I_1, the moment of inertia of the rug beater with the shorter handle:

$$I_1 = \tfrac{1}{3}(1.0\text{ kg})(1.0\text{ m})^2 + \tfrac{1}{12}(0.50\text{ kg})(0.40\text{ m})^2 + (0.50\text{ kg})\left(1.0\text{ m} + \tfrac{1}{2}(0.4\text{ m})\right)^2 = 1.06\text{ kg}\cdot\text{m}^2$$

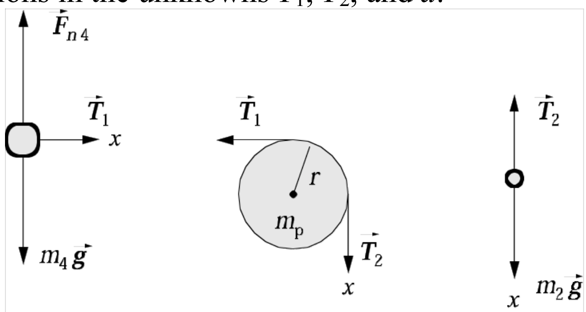
Substitute numerical values and evaluate I_2, the moment of inertia of the rug beater with the longer handle:

$$I_2 = \tfrac{1}{3}(1.5\text{ kg})(0.75\text{ m})^2 + \tfrac{1}{12}(0.60\text{ kg})(0.30\text{ m})^2 + (0.60\text{ kg})\left(0.75\text{ m} + \tfrac{1}{2}(0.3\text{ m})\right)^2 = 0.77\text{ kg}\cdot\text{m}^2$$

Because $I_2 < I_1$, the more massive rug beater will be easier to swing.

71 •• **[SSM]** The system shown in Figure 9-55consists of a 4.0-kg block resting on a frictionless horizontal ledge. This block is attached to a string that passes over a pulley, and the other end of the string is attached to a hanging 2.0-kg block. The pulley is a uniform disk of radius 8.0 cm and mass 0.60 kg. Find the acceleration of each block and the tension in the string.

Picture the Problem The diagrams show the forces acting on each of the masses and the pulley. We can apply Newton's 2nd law to the two blocks and the pulley to obtain three equations in the unknowns T_1, T_2, and a.



Apply Newton's 2nd law to the two blocks and the pulley:

$$\sum F_x = T_1 = m_4 a\,, \qquad (1)$$

$$\sum \boldsymbol{\tau}_p = (\boldsymbol{T}_2 - \boldsymbol{T}_1)\boldsymbol{r} = \boldsymbol{I}_p\boldsymbol{\alpha}\,, \qquad (2)$$

and

$$\sum F_x = m_2 g - T_2 = m_2 a \qquad (3)$$

Substitute for I_p and α in equation (2) to obtain:

$$T_2 - T_1 = \tfrac{1}{2}M_p a \qquad (4)$$

Eliminate T_1 and T_2 between equations (1), (3) and (4) and solve for a:

$$a = \frac{m_2 g}{m_2 + m_4 + \frac{1}{2}M_p}$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a} = \frac{(2.0\,\text{kg})(9.81\,\text{m/s}^2)}{2.0\,\text{kg} + 4.0\,\text{kg} + \frac{1}{2}(0.60\,\text{kg})} = 3.11\,\text{m/s}^2 = \boxed{3.1\,\text{m/s}^2}$$

Using equation (1), evaluate T_1:

$$\boldsymbol{T}_1 = (4.0\,\text{kg})(3.11\,\text{m/s}^2) = \boxed{12\,\text{N}}$$

Solve equation (3) for T_2:

$$T_2 = m_2(g - a)$$

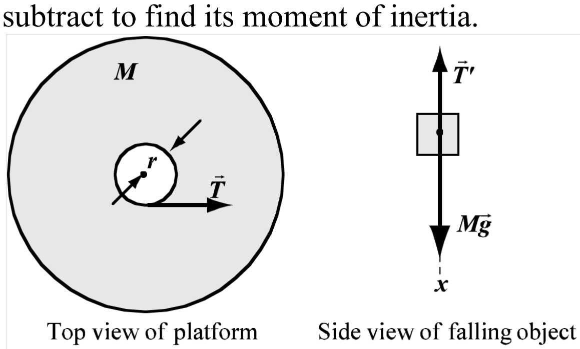
Substitute numerical values and evaluate T_2:

$$T_2 = (2.0\,\text{kg})(9.81\,\text{m/s}^2 - 3.11\,\text{m/s}^2) = \boxed{13\,\text{N}}$$

Remarks: Note that the only effect of the pulley is to change the direction of the force in the string.

82 •• A device for measuring the moment of inertia of an object is shown in Figure 9-63. The circular platform is attached to a concentric drum of radius R, and are free to rotate about a frictionless vertical axis. The string that is wound around the drum passes over a frictionless and massless pulley to a block of mass M. The block is released from rest, and the time t_1 it takes for it to drop a distance D is measured. The system is then rewound, the object whose moment of inertia I we wish to measure is placed on the platform, and the system is again released from rest. The time t_2 required for the block to drop the same distance D then provides the data needed to calculate I. Using R = 10 cm, M = 2.5 kg, D = 1.8 m and t_1 = 4.2 s and t_2 = 6.8 s, (*a*) find the moment of inertia of the platform-drum combination, (*b*) Using t_2 = 6.9 s, find the moment of inertia of the platform-drum-object combination. (*c*) Use your results for Parts (*a*) and (*b*) to find the moment of inertia of the object.

Picture the Problem Let r be the radius of the concentric drum (10 cm) and let I_0 be the moment of inertia of the drum plus platform. We can use Newton's 2nd law in both translational and rotational forms to express I_0 in terms of a and a constant-acceleration equation to express a and then find I_0. We can use the same equation to find the total moment of inertia when the object is placed on the platform and then subtract to find its moment of inertia.



(*a*) Apply Newton's 2nd law to the platform and the weight:

$$\sum \tau_0 = Tr = I_0\alpha \quad (1)$$

and

$$\sum F_x = Mg - T = Ma \quad (2)$$

Substitute a/r for α in equation (1) and solve for T:

$$T = \frac{I_0}{r^2}a$$

Substitute for T in equation (2) and solve for I_0 to obtain:

$$I_0 = \frac{Mr^2(g-a)}{a} \qquad (3)$$

Using a constant-acceleration equation, relate the distance of fall to the acceleration of the weight and the time of fall and solve for the acceleration:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$ and $\Delta x = D$,

$$a = \frac{2D}{(\Delta t)^2}$$

Substitute for a in equation (3) to obtain:

$$I_0 = Mr^2\left(\frac{g}{a} - 1\right) = Mr^2\left(\frac{g(\Delta t)^2}{2D} - 1\right)$$

Substitute numerical values and evaluate I_0:

$$I_0 = (2.5\,\text{kg})(0.10\,\text{m})^2\left[\frac{(9.81\,\text{m/s}^2)(4.2\,\text{s})^2}{2(1.8\,\text{m})} - 1\right] = 1.177\,\text{kg}\cdot\text{m}^2 = \boxed{1.2\,\text{kg}\cdot\text{m}^2}$$

(b) Relate the moments of inertia of the platform, drum, shaft, and pulley (I_0) to the moment of inertia of the object and the total moment of inertia:

$$I_{\text{tot}} = I_0 + I = Mr^2\left(\frac{g}{a} - 1\right) = Mr^2\left(\frac{g(\Delta t)^2}{2D} - 1\right)$$

Substitute numerical values and evaluate I_{tot}:

$$I_{\text{tot}} = (2.5\,\text{kg})(0.10\,\text{m})^2\left[\frac{(9.81\,\text{m/s}^2)(6.8\,\text{s})^2}{2(1.8\,\text{m})} - 1\right] = 3.125\,\text{kg}\cdot\text{m}^2 = \boxed{3.1\,\text{kg}\cdot\text{m}^2}$$

I is the difference between I_{tot} and I_0:

$$I = I_{\text{tot}} - I_0$$

Substitute numerical values and evaluate I:

$$I = 3.125\,\text{kg}\cdot\text{m}^2 - 1.177\,\text{kg}\cdot\text{m}^2 = \boxed{1.9\,\text{kg}\cdot\text{m}^2}$$

84 • An object is rolling without slipping. What percentage of its total kinetic energy is its translational kinetic energy if the object is (a) a uniform sphere, (b) a uniform cylinder, or (c) a hoop.

Picture the Problem The total kinetic energy of any object that is rolling without slipping is given by $K = K_{\text{trans}} + K_{\text{rot}}$. We can find the percentages associated with each motion by expressing the moment of inertia of the objects as kmr^2 and deriving a general expression for the ratios of rotational kinetic energy to total kinetic energy and translational kinetic energy to total kinetic energy and substituting the appropriate values of k.

Express the total kinetic energy associated with a rotating and translating object:

$$K = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(kmr^2\right)\frac{v^2}{r^2}$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}kmv^2 = \tfrac{1}{2}mv^2(1+k)$$

Express the ratio $\dfrac{K_{\text{trans}}}{K}$:

$$\frac{K_{\text{trans}}}{K} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2(1+k)} = \frac{1}{1+k}$$

(*a*) Substitute $k = \frac{2}{5}$ for a uniform sphere to obtain:

$$\left.\frac{K_{\text{trans}}}{K}\right|_{\text{sphere}} = \frac{1}{1+0.4} = 0.714 = \boxed{71.4\%}$$

(*b*) Substitute $k = \frac{1}{2}$ for a uniform cylinder to obtain:

$$\left.\frac{K_{\text{trans}}}{K}\right|_{\text{cylinder}} = \frac{1}{1+0.5} = 0.667 = \boxed{66.7\%}$$

(*c*) Substitute $k = 1$ for a hoop to obtain:

$$\left.\frac{K_{\text{trans}}}{K}\right|_{\text{hoop}} = \frac{1}{1+1} = 0.500 = \boxed{50.0\%}$$

92 •• A thin spherical shell and solid sphere of the same mass m and radius R roll without slipping down an incline through the same vertical drop H (Figure 9-64). Each is moving horizontally as it leaves the ramp. The spherical shell hits the ground a horizontal distance L from the end of the ramp and the solid sphere hits the ground a distance L' from the end of the ramp. Find the ratio L'/L.

Picture the Problem Let the zero of gravitational potential energy be at the elevation where the spheres leave the ramp. The distances the spheres will travel are directly proportional to their speeds when they leave the ramp.

Express the ratio of the distances traveled by the two spheres in terms of their speeds when they leave the ramp:

$$\frac{L'}{L} = \frac{v_{\text{solid}}\Delta t}{v_{\text{shell}}\Delta t} = \frac{v_{\text{solid}}}{v_{\text{shell}}} \qquad (1)$$

Use conservation of mechanical energy to find the speed of the spheres when they leave the ramp:

$$\Delta K + \Delta U = 0$$

or, because $K_{\text{i}} = U_{\text{f}} = 0$,

$$K_{\text{f}} - U_{\text{i}} = 0 \qquad (2)$$

Express K_{f} for the spheres and simplify to obtain (Note that $k = 2/3$ for the spherical shell and 2/5 for the uniform sphere):

$$K_{\text{f}} = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(kmR^2\right)\frac{v^2}{R^2}$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}kmv^2 = (1+k)\tfrac{1}{2}mv^2$$

Substitute for K_{f} in equation (2) to obtain:

$$(1+k)\tfrac{1}{2}mv^2 = mgH \Rightarrow v = \sqrt{\frac{2gH}{1+k}}$$

Substitute for v_{shell} and v_{solid} in equation (1) and simplify to obtain:

$$\frac{L'}{L} = \sqrt{\frac{1+k_{\text{shell}}}{1+k_{\text{solid}}}} = \sqrt{\frac{1+\frac{2}{3}}{1+\frac{2}{5}}} = \boxed{1.09}$$

108 •• The radius of a small playground merry-go-round is 2.2 m. To start it rotating, you wrap a rope around its perimeter and pull with a force of 260 N for 12 s. During this time, the merry-go-round makes one complete rotation. Neglect any effects of friction. (*a*) Find the angular acceleration of the merry-go-round. (*b*) What torque is exerted by the rope on the merry-go-round? (*c*) What is the moment of inertia of the merry-go-round?

Picture the Problem The force you exert on the rope results in a net torque that accelerates the merry-go-round. The moment of inertia of the merry-go-round, its angular acceleration, and the torque you apply are related through Newton's 2nd law.

(*a*) Using a constant-acceleration equation, relate the angular displacement of the merry-go-round to its angular acceleration and acceleration time:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2 \Rightarrow \boldsymbol{\alpha} = \frac{2\text{Ä}\boldsymbol{\theta}}{(\text{Ä}\boldsymbol{t})^2}$$

Substitute numerical values and evaluate α:

$$\boldsymbol{\alpha} = \frac{2(2\pi\ \text{rad})}{(12\ \text{s})^2} = 0.0873\ \text{rad/s}^2$$
$$= \boxed{0.087\ \text{rad/s}^2}$$

(*b*) Use the definition of torque to obtain:

$$\boldsymbol{\tau} = \boldsymbol{Fr} = (260\ \text{N})(2.2\ \text{m}) = 572\ \text{N}\cdot\text{m}$$
$$= \boxed{0.57\ \text{kN}\cdot\text{m}}$$

(*c*) Use Newton's 2nd law to find the moment of inertia of the merry-go-round:

$$\boldsymbol{I} = \frac{\boldsymbol{\tau}_{\text{net}}}{\boldsymbol{\alpha}} = \frac{572\,\text{N}\cdot\text{m}}{0.0873\,\text{rad/s}^2} = \boxed{6.6\times10^3\,\text{kg}\cdot\text{m}^2}$$

114 •• A day-care center has a merry-go-round that consists of a uniform 240-kg circular wooden platform 4.00 m in diameter. Four children run alongside the merry-go-round and push tangentially along the platform's circumference until, starting from rest, the merry-go-round is spinning at 2.14 rev/min. During the spin up: (*a*) If each child exerts a sustained force equal to 26 N how far does each child run? (*b*) What is the angular acceleration of the merry-go-round during spin up? (*c*) How much work does each child do? (*d*) What is the increase in the kinetic energy of the merry-go-round?

Picture the Problem The work done by the four children on the merry-go-round will change its kinetic energy. We can use the work-energy theorem to relate the work done by the children to the distance they ran and Newton's 2nd law to find the angular acceleration of the merry-go-round.

(*a*) Use the work-kinetic energy theorem to relate the work done by the children to the kinetic energy of the merry-go-round:

$$W_{\text{net force}} = \Delta K = K_{\text{f}}$$

or

$$4F\Delta s = \tfrac{1}{2}I\omega^2$$

Substitute for *I* to obtain:

$$4F\Delta s = \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\omega^2 = \tfrac{1}{4}mr^2\omega^2$$

Solving for Δs yields:

$$\Delta s = \frac{mr^2\omega^2}{16F}$$

Substitute numerical values and evaluate Δs:

$$\Delta s = \frac{(240\,\text{kg})(2.00\,\text{m})^2\left[\frac{1\,\text{rev}}{2.8\,\text{s}}\times\frac{2?\,\text{rad}}{\text{rev}}\right]^2}{16(26\,\text{N})} = 11.6\,\text{m} = \boxed{12\,\text{m}}$$

(*b*) Apply Newton's 2nd law to express the angular acceleration of the merry-go-round:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{4Fr}{\tfrac{1}{2}mr^2} = \frac{8F}{mr}$$

Substitute numerical values and evaluate α:

$$\boldsymbol{\alpha} = \frac{8(26\,\text{N})}{(240\,\text{kg})(2.00\,\text{m})} = \boxed{0.43\,\text{rad/s}^2}$$

(*c*) Use the definition of work to

$$W = F\Delta s = (26\,\text{N})(11.6\,\text{m}) = \boxed{0.30\,\text{kJ}}$$

relate the force exerted by each child to the distance over which that force is exerted:

(*d*) Relate the kinetic energy of the merry-go-round to the work that was done on it:

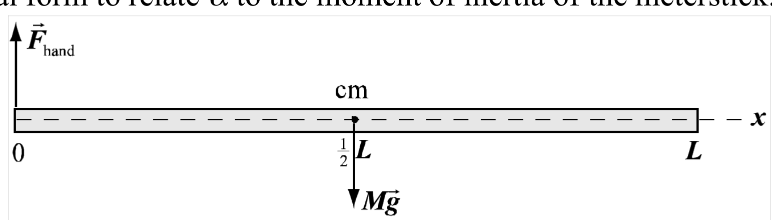
$$W_{\text{net force}} = \Delta K = K_f - 0 = 4F\Delta s$$

Substitute numerical values and evaluate $W_{\text{net force}}$:

$$W_{\text{net force}} = 4(26\,\text{N})(11.6\,\text{m}) = \boxed{1.2\,\text{kJ}}$$

121 •• **[SSM]** A popular classroom demonstration involves taking a meterstick and holding it horizontally at the 0.0-cm end with a number of pennies spaced evenly along its surface. If the hand is suddenly relaxed so that the meterstick pivots freely about the 0.0-cm mark under the influence of gravity, an interesting thing is seen during the first part of the stick's rotation: the pennies nearest the 0.0-cm mark remain on the meterstick, while those nearest the 100-cm mark are left behind by the falling meterstick. (This demonstration is often called the ″faster than gravity″ demonstration.) Suppose this demonstration is repeated without any pennies on the meterstick. (*a*) What would the initial acceleration of the 100.0-cm mark then be? (The initial acceleration is the acceleration just after the release.) (*b*) What point on the meterstick would then have an initial acceleration greater than *g*?

Picture the Problem The diagram shows the force the hand supporting the meterstick exerts at the pivot point and the force the earth exerts on the meterstick acting at the center of mass. We can relate the angular acceleration to the acceleration of the end of the meterstick using $a = L\alpha$ and use Newton's 2nd law in rotational form to relate α to the moment of inertia of the meterstick.



(*a*) Relate the acceleration of the far end of the meterstick to the angular acceleration of the meterstick:

$$a = L\alpha \qquad (1)$$

Apply $\sum \tau_P = I_P\alpha$ to the meterstick:

$$Mg\left(\frac{L}{2}\right) = I_P\alpha \Rightarrow \alpha = \frac{MgL}{2I_P}$$

From Table 9-1, for a rod pivoted at one end, we have:

$$I_P = \frac{1}{3}ML^2$$

Substitute for I_P in the expression for α to obtain:

$$\alpha = \frac{3MgL}{2ML^2} = \frac{3g}{2L}$$

Substitute for α in equation (1) to obtain:

$$a = \frac{3g}{2}$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a} = \frac{3\left(9.81\,\text{m/s}^2\right)}{2} = \boxed{14.7\,\text{m/s}^2}$$

(b) Express the acceleration of a point on the meterstick a distance x from the pivot point:

$$a = \alpha x = \frac{3g}{2L}x$$

Express the condition that the meterstick have an initial acceleration greater than g:

$$\frac{3g}{2L}x > g \Rightarrow \boldsymbol{x} > \frac{2\boldsymbol{L}}{3}$$

Substitute the numerical value of L and evaluate x:

$$\boldsymbol{x} > \frac{2\left(100.0\,\text{cm}\right)}{3} = \boxed{66.7\,\text{cm}}$$